Lesson 3: Developing Mathematical Empathy

by Vivian Shell

Grade Level(s)

12th grade students enrolled in “Mathematical Decision Making for Life”

Lesson Overview

Students will use an activity based on Mayan/Egyptian math to talk more concretely about place value and numeracy. This activity will help the class further develop the learner profile that was started in the previous lesson. They will use this information to develop interview questions for the next stage.

Learning Objectives

Students will analyze big ideas of place value while constructing their own arguments and critiquing the reasoning of fellow classmates in preparation for critiquing the reasoning of younger students.

Standards

- How to construct viable arguments and critique the reasoning of others. In particular, how to:
    - Use stated assumptions, definitions, and previously established results to construct an argument. (MDMFL IV.1a)
    - Recognize and use counterexamples. (MDMFL IV.1c)
    - Justify and communicate conclusions, and respond to the arguments of others. (MDMFL IV.1.d)
- How to evaluate a speaker’s point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used. (SL 11-12.3)

Preparation

- Work through the Mayan/Egyptian math assignment yourself.
- Be prepared with possible times for visiting the elementary school. ***(It is assumed that you have already established a relationship with a specific teacher or program and that it is known that you will be doing this design challenge with them!)***
- Be prepared with your own understanding of the connections between the Mayan/Egyptian math assignment and the Utah Elementary Math Core.

Materials and Resources

- Copies of the Utah Elementary Math Core Standards (or the relevant standards for your school)
- Copies of Mayan Math and Egyptian Math (one per student, printed back to back - below)
- Cut-out tags of Mayan numerals (one per student plus one per team, mixed up - below)
- Small post-its (many per student, enough for annotating the worksheets)
- Design Thinking graphic organizer to display (below)
- School calendars (for both your school and the elementary school)
- Quote from Plato to display (below)

Activity 1: Analysis of Mayan Math Game with Core (*45 minutes*)

- "We have used our own experiences along with the framing of the article on equity and the TED talk to help us begin to create a learner profile. Now, we will begin to look at some mathematics itself to find additional descriptors that are more specific to learners of mathematics."
- Give each student a set of cut-out tags of Mayan numerals. Their task is to arrange these symbols into whatever sequence they see fit. They should do this individually on their desks, to increase the number of ways that students approach this. (~5-7 minutes)
- Do a silent Museum Walk through the room so everyone can see how every other student chose to arrange the tags. Students should be looking for ways to describe what they see, as this will become important when they work with their elementary user.
- Organize students into their design teams and have them take their sequence with them. Ask them to observe the sequences of other members in their team and take turns restating the reasoning that was used until their team-mate is satisfied that his/her reasoning was understood. (~5-8 minutes)
- Pass-out the Mayan Math/Egyptian Math worksheet and have students work on this in teams. State the purpose of this activity: "As you work, please make note of the mathematical ideas you believe each problem is designed to represent. Talk, talk, talk about this with your team-mates." Circulate to answer questions and build your own empathy with the math abilities and level of metacognition of your students. (~10-15 minutes)

- Debrief the worksheet as a class. Go over “answers” and name the mathematical ideas that they uncovered as they worked. Write these ideas on small post-its that they leave on top of each problem for later use.
- “To push you toward developing academic language yourselves, I’d like you to take these observations of mathematical ideas and see if you can find them in the core standards for mathematics in elementary school.” Pass out copies of the core standards, one set per team to encourage collaboration, and allow time for them to cross-reference the class observations. They should write the code for the standard on the post-its. (~5-10 minutes)

Activity 2: Addition to Learner Profile (*10 minutes*)

- “Let’s challenge ourselves to add to our learner profile some specific descriptors of *learner characteristics in mathematics*. This is a challenge because I want you to describe a characteristic of a person but specifically from the stand point of the person as a mathematician! For example, I have observed that some students approach this assignment by looking for symmetry in how they arrange the tags. I will write ‘looks for symmetry’ and add it to our poster.”
- Ask students to stand around the learner profile poster from the previous lesson and give students a stack of post-its. Ask them to reflect on their mathematical insights from the Mayan/Egyptian Math activity and write down descriptors of learners as mathematicians. Allow 3-4 minutes for this silently, then ask students to share an idea on a post-it, one student at a time, and stick it to the poster. Encourage many post-its. They may write additional ideas as the share-out continues.

Activity 3: Introduction to Design Challenge (*10 minutes*)

- Display the design thinking graphic organizer as a reminder of the design thinking process. “Our challenge is to create a learning experience for a particular student at the elementary school in order to impact their outcome in mathematics. This will be our way of influencing equity in education while simultaneously becoming more sophisticated in our own understanding of mathematics and, hopefully, enjoy it more! Right now, let’s address the logistics of this challenge.”
- Display a school calendar for timing visits with the elementary school. You will need:
  - a day to visit and conduct the interview
  - a day or two to define and ideate after the interview
  - a day or two to prototype based on the ideation
  - a day to revisit the classroom and test the prototype
  - a day or two to reflect on how the prototype met the user’s needs and iterate on the design
  - a day to return to the elementary school to test the new prototype. Depending on your school calendar, this could cover a 2-3 week time period. List these needs for all to see and create a schedule for accomplishing these tasks.

Activity 4: Preparation for Interview (~*5-8 minutes*)

- Display quote about 'play' from Plato. "In your teams, you will now prepare for your interview with an elementary student. Recall that you will be designing a math activity to address a specific, individual need of that student. You will need to have empathy for him or her as a person, as a learner, and as a mathematician. To help you see this person through multiple lenses, I am placing the requirement that you use the Mayan Math sequencing activity as part of your interview. I have found that if I give this task to a child, allow them some time to work on it, and talk to them a little as a they work, I can gain a lot of insight into their way of thinking and their attitudes about themselves as learners and as mathematicians. I have also discovered that this activity takes on the feel of 'play' as described in this quote. You will learn a lot about your user by being mathematically playful!"
- Provide each team with a set of cut-out tags of Mayan numerals, which they are required to use as part of the interview, and instruct them to develop a list of interview questions that reflect the insights about learners that we have developed in our learner profile work. Instruct them to determine the roles for members of their group: 1 primary interviewer, 1 note taker for affective empathy, 1 note taker for mathematical empathy. (While they may all participate in the interview, it may be helpful to reduce the complexity of listening for each individual team-member by assigning these roles.)

Troubleshooting

- Students will not be familiar with the language of the core standards. This language is highly specialized, and understanding what the standards say in a practical sense may be helpful for students to connect with the Mayan/Egyptian math worksheet. For this reason, it may be helpful to refer to the curriculum guides for these standards at the Utah State Office of Education website: http://www.schools.utah.gov/CURR/mathelem/Core.aspx
- It may be a challenge for high school students to name learner characteristics specific to math. A few of the characteristics they may observe are: looks for symmetry, finds a way to chunk the tags, lines up the tags with a connecting thought between each pair, translates to a number, asks repeatedly if they are 'right', dives in and invents their own way of organizing, smiles when they realize it's a puzzle, frowns when they realize it's a puzzle. It may be helpful to provide some of these insights to get them started.

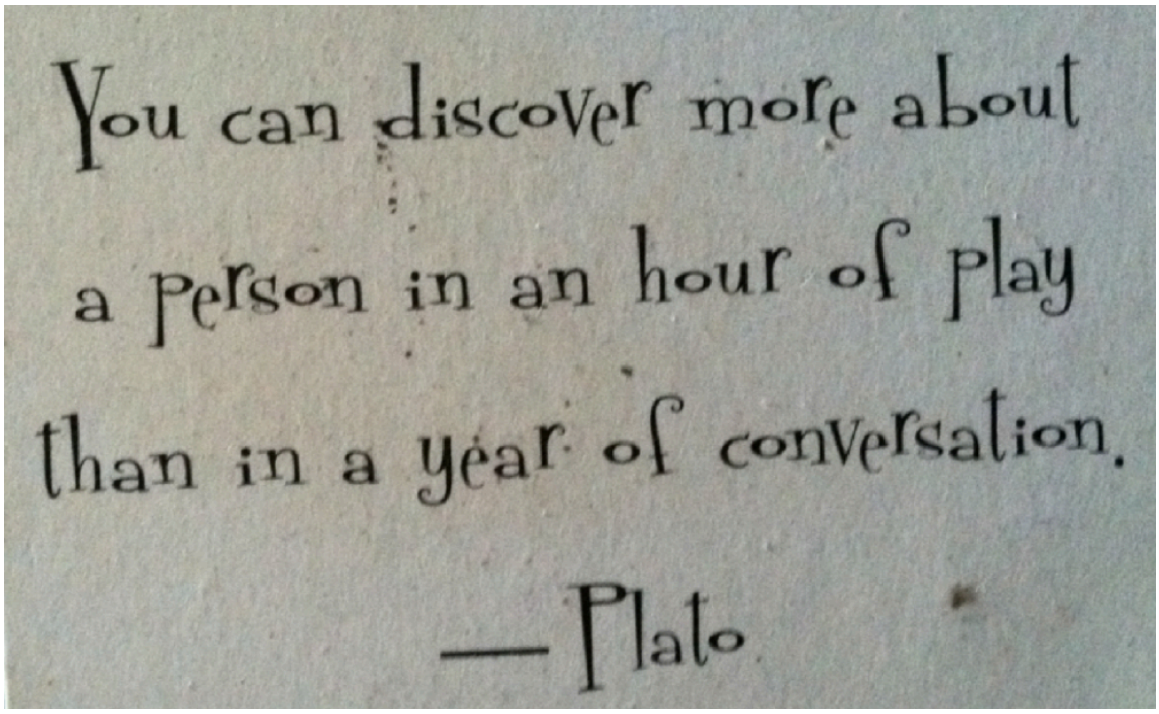
Assessment

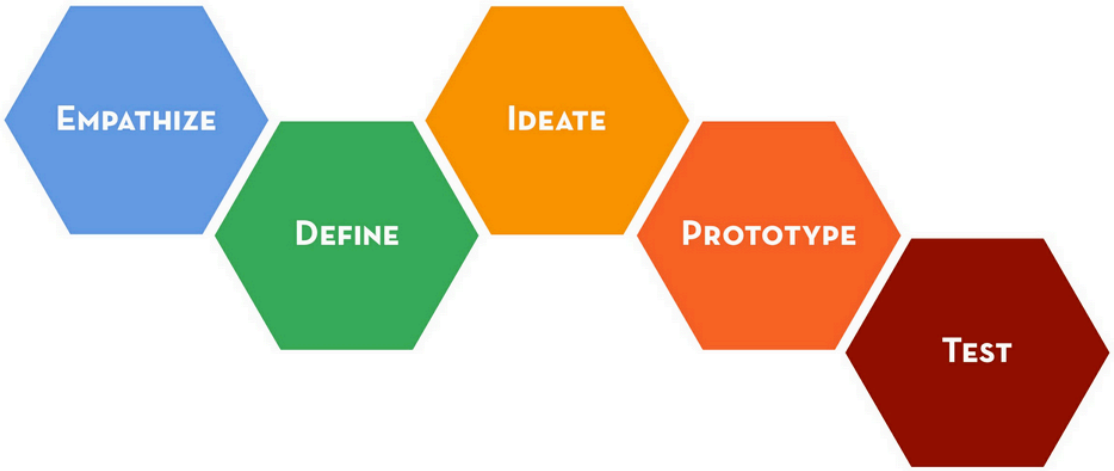
- Collect your own informal observations about the connections and understandings that your students display. These should include (but don't limit yourself!):
  - an understanding of regrouping
  - the role of order and the digit zero in place value
  - the difference between additive-types of naming numbers to naming numbers based on place value. (In Egyptian math, the order of symbols does not matter and our value is the result of adding the value of each of the symbols. In Mayan math, and in

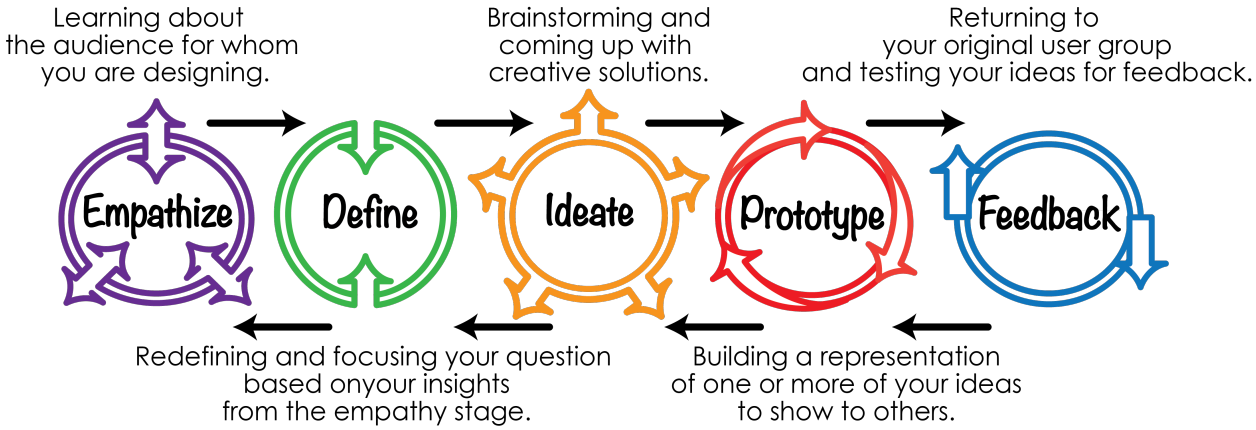
our current place value system, location of digits implies value and the addition of quantities by order of magnitude is implied)

  - the bridge from symbolic work that is additive to symbolic work that involves place value (This bridge occurs from the naming of 19 to the naming of 20 in Mayan math. In our Hindu-Arabic system, we do not have the additive element in our digits from 1-9, so this shift in thinking is not as easy to identify)
  - the different roles of number in mathematical problems that involve binary operations between two numbers (e.g. In problem 1 of Mayan Math, the first number, 25, can represent the number of objects that one starts with. The second number, 5, can represent the number of piles one should use to arrange these 25 objects. In problem 4, if a student literally removes the "dot" from twenty, the result would be zero. However, the problem demonstrates the abstractness and power of a place value system. We are not literally removing the "dot". We are to remove the *value* of the "dot" when it is located in a certain place.)
  - knowing the meaning of operation symbols and the order of operations

- Collect informal observations about the affective strengths of your students.
- Students' abilities to interpret the core curriculum are secondary in this activity, as this is most likely their first introduction to such specialized language. The purpose of this is to further develop their academic English. Make note of words and phrases that are unfamiliar to them or need unpacking and decide if they need to be cleared up now or at a later date.
- Be sure that students have generated a list of interview questions.


You can discover more about
a person in an hour of play
than in a year of conversation.
— Plato


Empathize
Define
Ideate
Prototype
Test


Learning about the audience for whom you are designing.
Brainstorming and coming up with creative solutions.
Returning to your original user group and testing your ideas for feedback.
Empathize
Define
Ideate
Prototype
Feedback
Redefining and focusing your question based onyour insights from the empathy stage.
Building a representation of one or more of your ideas to show to others.

Name _____

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Evaluate each expression using Mayan numbers.
Please show all work.

1. ≡ / ⎽

2. ⎽ + ⎽ + ⎽

3. 𝟎 – •

4. ═ + ••••̲ – ≡

5. ═ + •• * ••••

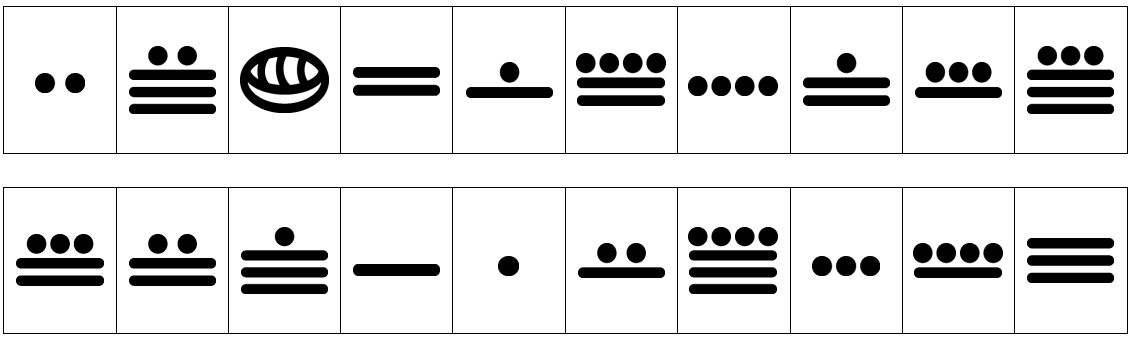
6. ••• (═̲ – ••̲)

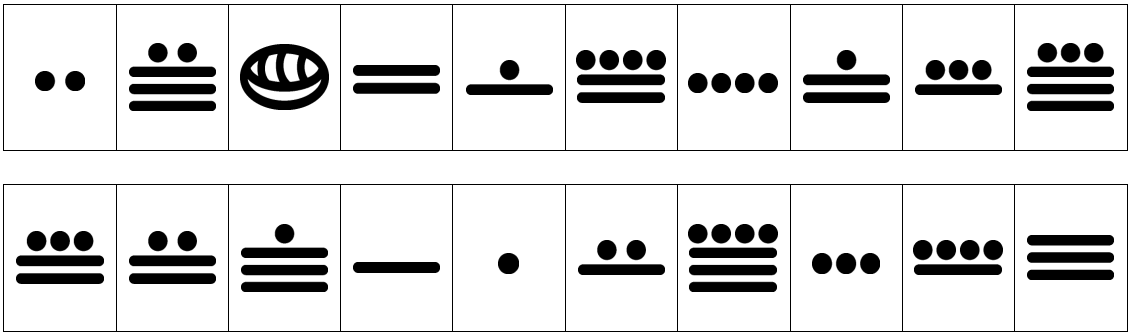
7. ≡ – (⎽ + ••)

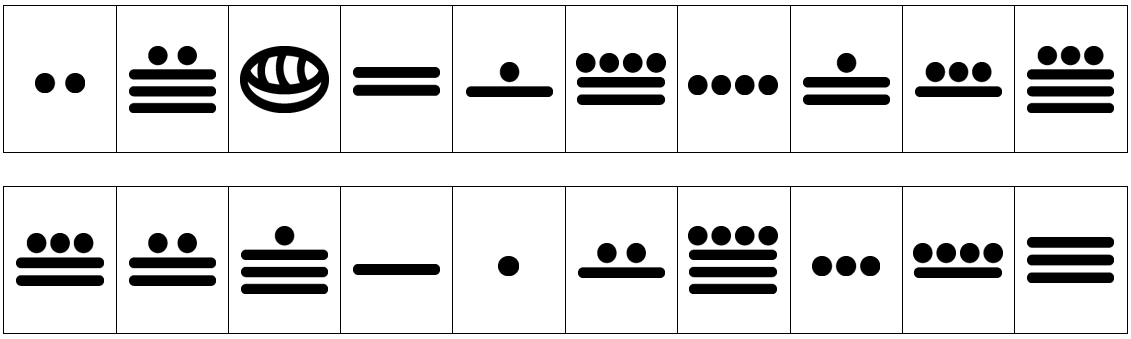
8. (≡̇ – ≡)^≡

☺ •••• (▬ – •) / •••̲ + ••••̲

Created by Diane Crim at Salt Lake Center for Science Education, Salt Lake City, Utah







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Name _____

Fill in the blanks with the missing number.

1.		
2.		**251**
3.		
4.		**300,000**
5.		
6.		
7.		**4,100,003**
8.		
9.		
10.		**2,080,530**

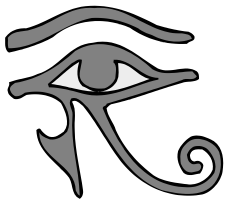
1	***tally***
10	***heel bone***
100	***coiled rope***
1,000	***lotus flower***
10,000	***bent stick***
100,000	***tadpole***
1,000,000	***astonished man***

11. Do you think that the Egyptian numbering system has a picture for zero?

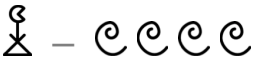
Do you think that they needed it? Explain. _____

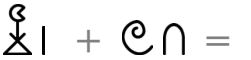
_____

Write each answer using Egyptian numbers.



12. = _____

13.  = _____

14.  = _____

15.  = _____

16.  / = _____

 = _____